Exercise 25

Use a linear approximation (or differentials) to estimate the given number.

$$\sqrt[3]{1001}$$

Solution

Compute the derivative of $y = \sqrt[3]{x}$.

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt[3]{x}$$

$$= \frac{d}{dx}(x^{1/3})$$

$$= \frac{1}{3}x^{-2/3}$$

Consequently, the differential of $y = \sqrt[3]{x}$ is

$$dy = \frac{1}{3}x^{-2/3}\,dx.$$

In order to estimate $\sqrt[3]{1001}$, set $x = 1000$ and $dx = 1$.

$$dy = \frac{1}{3}(1000)^{-2/3}(1) = \frac{1}{300}$$

Note that dy here is the vertical distance from the function's actual value at $x = 1000$ to the linear approximation's value at $x = 1001$.

$$\sqrt[3]{1001} \approx \sqrt[3]{1000} + \frac{1}{300} = 10 + \frac{1}{300} = \frac{3001}{300} = 10.00\bar{3}$$